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AM 1 * Utilizând, eventual, identitatea

$$\sin a - \sin b = 2 \sin \frac{a-b}{2} \cos \frac{a+b}{2},$$

să se calculeze

$$\lim_{x \to \infty} (\sin \sqrt{x+1} - \sin \sqrt{x}).$$

$$\lim_{x\to\infty}(\sin\sqrt{x+1}-\sin\sqrt{x}) = 2\cdot\lim_{x\to\infty}[\sin\frac{\sqrt{x+1}-\sqrt{x}}{2}\cos\frac{\sqrt{x+1}+\sqrt{x}}{2}] = 2\cdot 0\cdot\alpha = 0$$

b

$$\cos\frac{\sqrt{x+1}+\sqrt{x}}{2} = \alpha \in [-1;1]$$

$$\lim_{x\to\infty}\sin\frac{\sqrt{x+1}-\sqrt{x}}{2} = \lim_{x\to\infty}\sin\frac{\sqrt{x+1}-\sqrt{x}}{2}\cdot\frac{\sqrt{x+1}+\sqrt{x}}{\sqrt{x+1}+\sqrt{x}} = \lim_{x\to\infty}\sin\frac{x+1-x}{2(\sqrt{x+1}+\sqrt{x})} =$$

$$= \sin\lim_{x\to\infty}\frac{1}{2(\sqrt{x+1}+\sqrt{x})} = \sin 0 = 0$$

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AM 2 * Calculaţi

$$\lim_{x\to\infty} x^4\left(e^{\frac{1}{x^2+1}} - e^{\frac{1}{x^2}}\right).$$

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AM 3 * Calculaţi

$$\lim_{x\to\infty}\frac{\left[\frac{x}{4}\right]}{x}.$$

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AM 4 Să se calculeze:

$$\lim_{x\to\infty}(x^2 - x\ln(e^x+1)).$$

AM 5 Calculaţi

$$\lim_{x\to\infty}\left(\sqrt{x+1}-\sqrt{x}\right)\sqrt{x}\,.$$

$$\lim_{x\to\infty}(\sqrt{x+1}-\sqrt{x})\cdot\sqrt{x}=\lim_{x\to\infty}\frac{\sqrt{x+1}+\sqrt{x}}{x+1-x}\cdot\sqrt{x}=\lim_{x\to\infty}(\sqrt{x(x+1)}+x)=\infty$$

c

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AM 6 Se consideră funcţia $f:\mathbb{R}\to\mathbb{R}$, $f(x)=e^x-x-1$. Să se calculeze

$$\lim_{x\to\infty}\frac{f'(x)}{f''(x)}.$$

$$\lim_{x\to\infty}\frac{f'(x)}{f''(x)}=\lim_{x\to\infty}\frac{e^x-1}{e^x}=\lim_{x\to\infty}\left(1-\frac{1}{e^x}\right)=1$$

b

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AM 7 Calculaţi limita

$$\lim_{x\to\infty}\frac{e^x+\pi^x}{4^x}.$$

$$\lim_{x\to\infty}\frac{e^x-\pi^x}{4^x}=\lim_{x\to\infty}\frac{\pi^x(\frac{e^x}{\pi^x}-1)}{4^x}=\lim_{x\to\infty}\frac{\pi^x}{4^x}(\frac{e^x}{\pi^x}-1)=\lim_{x\to\infty}(\frac{\pi}{4})^x[(\frac{e}{\pi})^x-1]=0\cdot(0-1)=0$$

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AM 8 * Să se calculeze limita

$$\lim_{x\to\infty}\left[1-\left(\frac{e}{\pi}\right)^x\right].$$

$$\lim_{x\to\infty}[1-(\frac{e}{\pi})^x]=1-0=1$$

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AM 9 Se consideră $f:(-1,+\infty)\to\mathbb{R}$, $f(x)=\ln(1+x)$. Atunci

$$\lim_{x\to 0}\frac{f(x)-f(0)}{x}$$

este:

$$\lim_{x\to 0}\frac{f(x)-f(0)}{x}=\lim_{x\to 0}\frac{\ln(1+x)-\ln 1}{x}=\lim_{x\to 0}\frac{1}{x}\ln(1+x)=\lim_{x\to 0}\ln(1+x)^{\frac{1}{x}}=\ln e=1$$

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AM 10 Calculaţi limita

$$\lim_{x\to\infty} \frac{x^4+1}{x^2+2} \ln \frac{x^2+1}{x^2}.$$

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AM 11 Să se calculeze:

$$\lim_{x \to -\infty} \left(x - \frac{x^3}{6} - \sin x \right).$$

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AM 12 Fie funcţia $f : \mathbb{R} \to \mathbb{R}$, definită prin $f(x) = \dfrac{x^{2013}}{2013^x}$. Calculaţi $\lim\limits_{x\to+\infty} f(x)$.

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty}\frac{x^{2013}}{2013^x} = \lim_{x\to\infty}\frac{2013\cdot x^{2012}}{2013^x\cdot\ln 2013} = \lim_{x\to\infty}\frac{2013\cdot 2012\cdot x^{2012}}{2013^x\cdot\ln 2012\cdot\ln 2013} = \ldots =$$

$$= \frac{2013\cdot 2012\cdot\ldots\cdot 1}{\ln 2013\cdot\ln 2013\cdot\ldots\cdot\ln 2013}\cdot\frac{1}{2013^x} = 0$$

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AM 13 Calculaţi $\lim\limits_{x\to\infty}(f(x))^x$, unde $f:\mathbb{R}-\{-1,1\}\to\mathbb{R}$,

$$f(x)=\frac{x^2+2x+5}{x^2-1}.$$

$$\lim_{x\to\infty}(f(x))^x=\lim_{x\to\infty}\left(\frac{x^2+2x+5}{x^2-1}\right)^x=\lim_{x\to\infty}\left(1+\frac{x^2+2x+5}{x^2-1}-1\right)^x=\lim_{x\to\infty}\left(1+\frac{2x+6}{x^2-1}\right)^{\frac{x^2-1}{2x+6}\cdot\frac{2x+6}{x^2-1}\cdot x}=$$

$$=e^{\lim\limits_{x\to\infty}\frac{2x+6}{x^2-1}\cdot x}=e^2$$

d

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AM 14 * Se consideră funcţia $f : \mathbb{R} \to \mathbb{R}$, $f(x) = \dfrac{2x^3}{x^2+1}$. Să se calculeze

$$\lim_{x\to\infty} \left(f(e^x)\right)^{\frac{1}{x}}.$$

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AM 15 * Fie $f:\mathbb{R}\to\mathbb{R}$, $f(x)=2x^4+x^2+3$. Să se calculzeze $\lim\limits_{x\to-\infty}\frac{f(x)}{x^4}$.

$$\lim_{x\to-\infty}\frac{f(x)}{x^4}=\lim_{x\to-\infty}\frac{2x^4+x^2+3}{x^4}=2$$

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AM 16 * Calculaţi următoarea limită $\lim_{x\to+\infty}(\sqrt{x+2}-\sqrt{x+1})$.

$$\lim_{x\to\infty}(\sqrt{x+2}-\sqrt{x+1})=\lim_{x\to\infty}\frac{x+2-x-1}{\sqrt{x+2}+\sqrt{x+1}}=\lim_{x\to\infty}\frac{1}{\sqrt{x+2}+\sqrt{x+1}}=\frac{1}{\infty}=0$$

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AM 17 Calculaţi următoarea limită $\lim\limits_{x\to+\infty}\left(\frac{x}{x+2}\right)^x$.

$$\lim_{x\to\infty}\left(\frac{x}{x+2}\right)^x=\lim_{x\to\infty}\left(1+\frac{x}{x+2}-1\right)^x=\lim_{x\to\infty}\left(1+\frac{-2}{x+2}\right)^{\frac{x+2}{-2}\cdot\frac{-2}{x+2}\cdot x}=e^{-2}=\frac{1}{e^2}$$

f

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AM 18 Fie funcţia $f:(0,+\infty)\to\mathbb{R}$, definită prin $f(x)=(2x^3+1)\ln x$.
Calculaţi $\lim\limits_{x\to+\infty}\frac{f(x)}{x^4}$.

$$\lim_{x\to\infty}\frac{f(x)}{x^4}=\lim_{x\to\infty}\frac{(2x^3+1)\ln x}{x^4}=\lim_{x\to\infty}\frac{2x^3+1}{x^3}\cdot\frac{\ln x}{x}=2\lim_{x\to\infty}\frac{\ln x}{x}=2\cdot 0=0$$

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AM 19 * Se consideră funcţia $f : (-2, 2) \to \mathbb{R}$, $f(x) = \ln \frac{2+x}{2-x}$. Să se calculeze

$$\lim_{x\to\infty} x f\left(\frac{1}{x}\right).$$

$$x \cdot f\left(\frac{1}{x}\right) = x \cdot \ln \frac{2+\frac{1}{x}}{2-\frac{1}{x}} = x \cdot \ln \frac{2x+1}{2x-1} = \ln\left(\frac{2x+1}{2x-1}\right)^x = \ln\left(1+\frac{2x+1}{2x-1}-1\right)^x = \ln\left(1+\frac{2}{2x-1}\right)^x$$

$$\lim_{x\to\infty} \ln\left(1+\frac{2}{2x-1}\right)^{\frac{2x-1}{2}\cdot\frac{2}{2x-1}\cdot x} = \ln e^{\lim\limits_{x\to\infty}\frac{2x}{2x-1}} = \ln e^{\frac{2}{2}} = 1$$

b

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AM 20 Fie $f : \mathbb{R} \to \mathbb{R}$,

$$f(x) = \frac{x^4+1}{(2x^2+1)(3+5x^2)} + \frac{2+x^2}{\sqrt{(4x^2+1)(9x^2+7)}}.$$

Calculaţi $\lim\limits_{x\to\infty} f(x)$.

$$\lim_{x\to\infty}\left[\frac{x^4+1}{(2x^2+1)(3+5x^2)} + \frac{2+x^2}{\sqrt{(4x^2+1)(9x^2+7)}}\right] = \frac{1}{2\cdot 5} + \frac{1}{\sqrt{4\cdot 9}} = \frac{1}{10} + \frac{1}{6} =$$

$$= \frac{3+5}{30} = \frac{8}{30} = \frac{4}{15}$$

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AM 21 * Calculaţi

$$L = \lim_{x \to 0} \frac{x^2 \sin\left(\frac{1}{x}\right)}{\sin x}.$$

$$L = \lim_{x \to 0} \frac{x^2 \cdot \sin \frac{1}{x}}{\sin x} = \lim_{x \to 0} \frac{x}{\sin x} \cdot (x \cdot \sin \frac{1}{x}) = 1 \cdot 0 \cdot \alpha = 0$$

$$\sin \frac{1}{x} = \alpha \in [-1;1]$$

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AM 22 * Fie funcţia $f : [0,2] \to \mathbb{R}$, $f(x) = \{x\}(1-\{x\})^2$, unde $\{x\}$ este partea fracţionară a lui x. Să se determine $\lim\limits_{x\to 1} f(x)$.

$$f(x)=\{x\}(1-\{x\})^2=\{x\}(1-2\{x\}+\{x\}^2)=\{x\}-2\{x\}^2+\{x\}^3=$$

$$=x-[x]-2(x-[x])^2+(x-[x])^3=x-[x]-2x^2+4x[x]-2[x]^2+x^3-$$

$$-3x^2[x]+3x[x]^2-[x]^3$$

$$f(x)=\begin{cases}0 & \textit{dacă} \quad x\in[0;1)\\ 1 & \textit{dacă} \quad x\in[1;2)\\ 2 & \textit{dacă} \quad x=2\end{cases}$$

$$\lim_{\substack{x\to 1\\ x<1}} f(x)=\lim_{\substack{x\to 1\\ x<1}}(x-2x^2+x^3)=1-2+1=0$$

$$\lim_{\substack{x\to 1\\ x>1}} f(x)=\lim_{\substack{x\to 1\\ x>1}}(x-1-2x^2+4x-2+x^3-3x^2+3x-1)=\lim_{\substack{x\to 1\\ x>1}}(x^3-5x^2+8x-4)=$$

$$=1-5+8-4=0$$

$$\Rightarrow \lim_{x\to 1} f(x)=0$$

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AM 23 Să se calculeze

$$\lim_{x\to 0} x \sin \frac{1}{x}.$$

$$\lim_{x\to 0}\left(x \cdot \sin \frac{1}{x}\right) = 0 \cdot \alpha = 0$$

$$\sin \frac{1}{x} = \alpha \in [-1;1]$$

b

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AM 24 * Fie funcţia $f : \mathbb{R}^* \to \mathbb{R}$, definită prin $f(x) = \frac{x}{\sqrt{x^2}}$. Calculaţi $\lim_{x \to 0} f(x)$.

$$\lim_{\substack{x \to 0 \\ x<0}} \frac{x}{\sqrt{x^2}} = \lim_{\substack{x \to 0 \\ x<0}} \frac{x}{|x|} = \lim_{\substack{x \to 0 \\ x<0}} \frac{x}{-x} = -\lim_{\substack{x \to 0 \\ x<0}} 1 = -1$$

$$\lim_{\substack{x \to 0 \\ x>0}} \frac{x}{\sqrt{x^2}} = \lim_{\substack{x \to 0 \\ x>0}} \frac{x}{|x|} = \lim_{\substack{x \to 0 \\ x>0}} \frac{x}{x} = \lim_{\substack{x \to 0 \\ x>0}} 1 = 1$$

$\Rightarrow$ *nu există* $\lim_{x \to 0} \frac{x}{\sqrt{x^2}}$

f

$$\sqrt{x^2} = |x|$$

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AM 25 Se consideră funcţia $f:\mathbb{R}\to\mathbb{R}$, definită prin $f(x)=\ln(1+x^2)$.
Calculaţi

$$\lim_{x\to 1}\frac{f(x)-f(1)}{x-1}.$$

$$\lim_{x\to 1}\frac{f(x)-f(1)}{x-1}=\lim_{x\to 1}\frac{\ln(1+x^2)-\ln 2}{x-1}\underset{\frac{0}{0}}{=}\lim_{x\to 1}\frac{1}{x-1}\ln\frac{1+x^2}{2}=\lim_{x\to 1}\ln\left(\frac{1+x^2}{2}\right)^{\frac{1}{x-1}}=$$

$$=\ln\lim_{x\to 1}\left(1+\frac{1+x^2}{2}-1\right)^{\frac{1}{x-1}}=\ln\lim_{x\to 1}\left(1+\frac{x^2-1}{2}\right)^{\frac{2}{x^2-1}\cdot\frac{x^2-1}{2}\cdot\frac{1}{x-1}}=\ln e^{\lim\limits_{x\to 1}\frac{x^2-1}{2(x-1)}}=\ln e^{\lim\limits_{x\to 1}\frac{x+1}{2}}=$$

$$=\ln e^{\frac{1+1}{2}}=\ln e=1$$

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AM 26 * Să se calculeze limita $\lim\limits_{x \to 0} \dfrac{\operatorname{tg} x}{x}$.

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AM 27 * Să se calculeze

$$\lim_{x \to 0} \sin \frac{1}{x}.$$

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AM 28 * Fie funcţia $f : (0, +\infty) \to \mathbb{R}$, definită prin $f(x) = x^x$. Calculaţi

$$\lim_{x \searrow 0} x^x .$$

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AM 29 * Fie $n \in \mathbb{N}^*$ fixat. Să se calculeze

$$\lim_{x \to 1} \frac{x + x^2 + \ldots + x^n - n}{x - 1}.$$

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