



AM 1 * Utilizând, eventual, identitatea

$$\sin a - \sin b = 2 \sin \frac{a-b}{2} \cos \frac{a+b}{2},$$

să se calculeze

$$\lim_{x \rightarrow \infty} (\sin \sqrt{x+1} - \sin \sqrt{x}).$$

$$\lim_{x \rightarrow \infty} (\sin \sqrt{x+1} - \sin \sqrt{x}) = 2 \cdot \lim_{x \rightarrow \infty} \left[\sin \frac{\sqrt{x+1} - \sqrt{x}}{2} \cos \frac{\sqrt{x+1} + \sqrt{x}}{2} \right] = 2 \cdot 0 \cdot \alpha = 0$$

b

$$\cos \frac{\sqrt{x+1} + \sqrt{x}}{2} = \alpha \in [-1; 1]$$

$$\lim_{x \rightarrow \infty} \sin \frac{\sqrt{x+1} - \sqrt{x}}{2} = \lim_{x \rightarrow \infty} \sin \frac{\sqrt{x+1} - \sqrt{x}}{2} \cdot \frac{\sqrt{x+1} + \sqrt{x}}{\sqrt{x+1} + \sqrt{x}} = \lim_{x \rightarrow \infty} \sin \frac{x+1-x}{2(\sqrt{x+1} + \sqrt{x})} =$$

$$= \sin \lim_{x \rightarrow \infty} \frac{1}{2(\sqrt{x+1} + \sqrt{x})} = \sin 0 = 0$$



AM 2 * Calculați

$$\lim_{x \rightarrow \infty} x^4 \left(e^{\frac{1}{x^2+1}} - e^{\frac{1}{x^2}} \right).$$



AM 3 * Calculați

$$\lim_{x \rightarrow \infty} \frac{\left[\frac{x}{4} \right]}{x}.$$



AM 4 Să se calculeze:

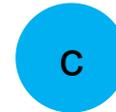
$$\lim_{x \rightarrow \infty} (x^2 - x \ln(e^x + 1)).$$



AM 5 Calculați

$$\lim_{x \rightarrow \infty} (\sqrt{x+1} - \sqrt{x})\sqrt{x}.$$

$$\lim_{x \rightarrow \infty} (\sqrt{x+1} - \sqrt{x}) \cdot \sqrt{x} = \lim_{x \rightarrow \infty} \frac{\sqrt{x+1} + \sqrt{x}}{x+1-x} \cdot \sqrt{x} = \lim_{x \rightarrow \infty} (\sqrt{x(x+1)} + x) = \infty$$





AM 6 Se consideră funcția $f : \mathbb{R} \rightarrow \mathbb{R}$, $f(x) = e^x - x - 1$. Să se calculeze

$$\lim_{x \rightarrow \infty} \frac{f'(x)}{f''(x)}.$$

$$\lim_{x \rightarrow \infty} \frac{f'(x)}{f''(x)} = \lim_{x \rightarrow \infty} \frac{e^x - 1}{e^x} = \lim_{x \rightarrow \infty} \left(1 - \frac{1}{e^x}\right) = 1$$

b



AM 7 Calculați limita

$$\lim_{x \rightarrow \infty} \frac{e^x + \pi^x}{4^x}$$

$$\lim_{x \rightarrow \infty} \frac{e^x - \pi^x}{4^x} = \lim_{x \rightarrow \infty} \frac{\pi^x \left(\frac{e^x}{\pi^x} - 1 \right)}{4^x} = \lim_{x \rightarrow \infty} \frac{\pi^x}{4^x} \left(\frac{e^x}{\pi^x} - 1 \right) = \lim_{x \rightarrow \infty} \left(\frac{\pi}{4} \right)^x \left[\left(\frac{e}{\pi} \right)^x - 1 \right] = 0 \cdot (0 - 1) = 0$$

d



AM 8 * Să se calculeze limita

$$\lim_{x \rightarrow \infty} \left[1 - \left(\frac{e}{\pi} \right)^x \right].$$

$$\lim_{x \rightarrow \infty} \left[1 - \left(\frac{e}{\pi} \right)^x \right] = 1 - 0 = 1$$





AM 9 Se consideră $f : (-1, +\infty) \rightarrow \mathbb{R}$, $f(x) = \ln(1 + x)$. Atunci

$$\lim_{x \rightarrow 0} \frac{f(x) - f(0)}{x}$$

este:

$$\lim_{x \rightarrow 0} \frac{f(x) - f(0)}{x} = \lim_{x \rightarrow 0} \frac{\ln(1 + x) - \ln 1}{x} = \lim_{x \rightarrow 0} \frac{1}{x} \ln(1 + x) = \lim_{x \rightarrow 0} \ln(1 + x)^{\frac{1}{x}} = \ln e = 1$$

b



AM 10 Calculați limita

$$\lim_{x \rightarrow \infty} \frac{x^4 + 1}{x^2 + 2} \ln \frac{x^2 + 1}{x^2}.$$



AM 11 Să se calculeze:

$$\lim_{x \rightarrow -\infty} \left(x - \frac{x^3}{6} - \sin x \right).$$



AM 12 Fie funcția $f : \mathbb{R} \rightarrow \mathbb{R}$, definită prin $f(x) = \frac{x^{2013}}{2013^x}$. Calculați $\lim_{x \rightarrow +\infty} f(x)$.

$$\begin{aligned} \lim_{x \rightarrow \infty} f(x) &= \lim_{x \rightarrow \infty} \frac{x^{2013}}{2013^x} = \lim_{x \rightarrow \infty} \frac{2013 \cdot x^{2012}}{2013^x \cdot \ln 2013} = \lim_{x \rightarrow \infty} \frac{2013 \cdot 2012 \cdot x^{2011}}{2013^x \cdot \ln 2012 \cdot \ln 2013} = \dots = \\ &= \frac{2013 \cdot 2012 \cdot \dots \cdot 1}{\ln 2013 \cdot \ln 2013 \cdot \dots \cdot \ln 2013} \cdot \frac{1}{2013^x} = 0 \end{aligned}$$

f



AM 13 Calculați $\lim_{x \rightarrow \infty} (f(x))^x$, unde $f : \mathbb{R} - \{-1, 1\} \rightarrow \mathbb{R}$,

$$f(x) = \frac{x^2 + 2x + 5}{x^2 - 1}.$$

$$\begin{aligned} \lim_{x \rightarrow \infty} (f(x))^x &= \lim_{x \rightarrow \infty} \left(\frac{x^2 + 2x + 5}{x^2 - 1} \right)^x = \lim_{x \rightarrow \infty} \left(1 + \frac{x^2 + 2x + 5}{x^2 - 1} - 1 \right)^x = \lim_{x \rightarrow \infty} \left(1 + \frac{2x + 6}{x^2 - 1} \right)^{\frac{x^2 - 1}{2x + 6} \cdot \frac{2x + 6}{x^2 - 1} \cdot x} = \\ &= e^{\lim_{x \rightarrow \infty} \frac{2x + 6}{x^2 - 1} \cdot x} = e^2 \end{aligned}$$

d



AM 14 * Se consideră funcția $f : \mathbb{R} \rightarrow \mathbb{R}$, $f(x) = \frac{2x^3}{x^2 + 1}$. Să se calculeze

$$\lim_{x \rightarrow \infty} \left(f(e^x) \right)^{\frac{1}{x}}.$$



AM 15 * Fie $f : \mathbb{R} \rightarrow \mathbb{R}$, $f(x) = 2x^4 + x^2 + 3$. Să se calculeze $\lim_{x \rightarrow -\infty} \frac{f(x)}{x^4}$.

$$\lim_{x \rightarrow -\infty} \frac{f(x)}{x^4} = \lim_{x \rightarrow -\infty} \frac{2x^4 + x^2 + 3}{x^4} = 2$$

a



AM 16 * Calculați următoarea limită $\lim_{x \rightarrow +\infty} (\sqrt{x+2} - \sqrt{x+1})$.

$$\lim_{x \rightarrow \infty} (\sqrt{x+2} - \sqrt{x+1}) = \lim_{x \rightarrow \infty} \frac{x+2-x-1}{\sqrt{x+2} + \sqrt{x+1}} = \lim_{x \rightarrow \infty} \frac{1}{\sqrt{x+2} + \sqrt{x+1}} = \frac{1}{\infty} = 0$$

c



AM 17 Calculați următoarea limită $\lim_{x \rightarrow +\infty} \left(\frac{x}{x+2} \right)^x$.

$$\lim_{x \rightarrow \infty} \left(\frac{x}{x+2} \right)^x = \lim_{x \rightarrow \infty} \left(1 + \frac{x}{x+2} - 1 \right)^x = \lim_{x \rightarrow \infty} \left(1 + \frac{-2}{x+2} \right)^{\frac{x+2}{-2} \cdot \frac{-2}{x+2} \cdot x} = e^{-2} = \frac{1}{e^2}$$

f



AM 18 Fie funcția $f : (0, +\infty) \rightarrow \mathbb{R}$, definită prin $f(x) = (2x^3 + 1) \ln x$.

Calculați $\lim_{x \rightarrow +\infty} \frac{f(x)}{x^4}$.

$$\lim_{x \rightarrow \infty} \frac{f(x)}{x^4} = \lim_{x \rightarrow \infty} \frac{(2x^3 + 1) \ln x}{x^4} = \lim_{x \rightarrow \infty} \frac{2x^3 + 1}{x^3} \cdot \frac{\ln x}{x} = 2 \lim_{x \rightarrow \infty} \frac{\ln x}{x} = 2 \cdot 0 = 0$$

e



AM 19 * Se consideră funcția $f : (-2, 2) \rightarrow \mathbb{R}$, $f(x) = \ln \frac{2+x}{2-x}$. Să se calculeze

$$\lim_{x \rightarrow \infty} x f\left(\frac{1}{x}\right).$$

$$x \cdot f\left(\frac{1}{x}\right) = x \cdot \ln \frac{2 + \frac{1}{x}}{2 - \frac{1}{x}} = x \cdot \ln \frac{2x + 1}{2x - 1} = \ln \left(\frac{2x + 1}{2x - 1}\right)^x = \ln \left(1 + \frac{2x + 1}{2x - 1} - 1\right)^x = \ln \left(1 + \frac{2}{2x - 1}\right)^x$$

$$\lim_{x \rightarrow \infty} \ln \left(1 + \frac{2}{2x - 1}\right)^{\frac{2x-1}{2} \cdot \frac{2}{2x-1} \cdot x} = \ln e^{\lim_{x \rightarrow \infty} \frac{2x}{2x-1}} = \ln e^{\frac{2}{2}} = 1$$

b



AM 20 Fie $f : \mathbb{R} \rightarrow \mathbb{R}$,

$$f(x) = \frac{x^4 + 1}{(2x^2 + 1)(3 + 5x^2)} + \frac{2 + x^2}{\sqrt{(4x^2 + 1)(9x^2 + 7)}}.$$

Calculați $\lim_{x \rightarrow \infty} f(x)$.

$$\begin{aligned} \lim_{x \rightarrow \infty} \left[\frac{x^4 + 1}{(2x^2 + 1)(3 + 5x^2)} + \frac{2 + x^2}{\sqrt{(4x^2 + 1)(9x^2 + 7)}} \right] &= \frac{1}{2 \cdot 5} + \frac{1}{\sqrt{4 \cdot 9}} = \frac{1}{10} + \frac{1}{6} = \\ &= \frac{3 + 5}{30} = \frac{8}{30} = \frac{4}{15} \end{aligned}$$

C



AM 21 * Calculați

$$L = \lim_{x \rightarrow 0} \frac{x^2 \sin\left(\frac{1}{x}\right)}{\sin x}.$$

$$L = \lim_{x \rightarrow 0} \frac{x^2 \cdot \sin \frac{1}{x}}{\sin x} = \lim_{x \rightarrow 0} \frac{x}{\sin x} \cdot \left(x \cdot \sin \frac{1}{x}\right) = 1 \cdot 0 \cdot \alpha = 0$$

$$\sin \frac{1}{x} = \alpha \in [-1; 1]$$

a



AM 22 * Fie funcția $f : [0, 2] \rightarrow \mathbb{R}$, $f(x) = \{x\}(1 - \{x\})^2$, unde $\{x\}$ este partea fracționară a lui x . Să se determine $\lim_{x \rightarrow 1} f(x)$.

$$\begin{aligned} f(x) &= \{x\}(1 - \{x\})^2 = \{x\}(1 - 2\{x\} + \{x\}^2) = \{x\} - 2\{x\}^2 + \{x\}^3 = \\ &= x - [x] - 2(x - [x])^2 + (x - [x])^3 = x - [x] - 2x^2 + 4x[x] - 2[x]^2 + x^3 - \\ &\quad - 3x^2[x] + 3x[x]^2 - [x]^3 \end{aligned}$$

$$f(x) = \begin{cases} 0 & \text{dacă } x \in [0;1) \\ 1 & \text{dacă } x \in [1;2) \\ 2 & \text{dacă } x = 2 \end{cases}$$

$$\lim_{\substack{x \rightarrow 1 \\ x < 1}} f(x) = \lim_{\substack{x \rightarrow 1 \\ x < 1}} (x - 2x^2 + x^3) = 1 - 2 + 1 = 0$$

$$\begin{aligned} \lim_{\substack{x \rightarrow 1 \\ x > 1}} f(x) &= \lim_{\substack{x \rightarrow 1 \\ x > 1}} (x - 1 - 2x^2 + 4x - 2 + x^3 - 3x^2 + 3x - 1) = \lim_{\substack{x \rightarrow 1 \\ x > 1}} (x^3 - 5x^2 + 8x - 4) = \\ &= 1 - 5 + 8 - 4 = 0 \end{aligned}$$

$$\Rightarrow \lim_{x \rightarrow 1} f(x) = 0$$

a



AM 23 Să se calculeze

$$\lim_{x \rightarrow 0} x \sin \frac{1}{x}$$

$$\lim_{x \rightarrow 0} \left(x \cdot \sin \frac{1}{x} \right) = 0 \cdot \alpha = 0$$

$$\sin \frac{1}{x} = \alpha \in [-1; 1]$$

b



AM 24 * Fie funcția $f : \mathbb{R}^* \rightarrow \mathbb{R}$, definită prin $f(x) = \frac{x}{\sqrt{x^2}}$. Calculați $\lim_{x \rightarrow 0} f(x)$.

$$\lim_{\substack{x \rightarrow 0 \\ x < 0}} \frac{x}{\sqrt{x^2}} = \lim_{\substack{x \rightarrow 0 \\ x < 0}} \frac{x}{|x|} = \lim_{\substack{x \rightarrow 0 \\ x < 0}} \frac{x}{-x} = -\lim_{\substack{x \rightarrow 0 \\ x < 0}} 1 = -1$$

$$\lim_{\substack{x \rightarrow 0 \\ x > 0}} \frac{x}{\sqrt{x^2}} = \lim_{\substack{x \rightarrow 0 \\ x > 0}} \frac{x}{|x|} = \lim_{\substack{x \rightarrow 0 \\ x > 0}} \frac{x}{x} = \lim_{\substack{x \rightarrow 0 \\ x > 0}} 1 = 1$$

nu există $\lim_{x \rightarrow 0} \frac{x}{\sqrt{x^2}}$



$$\sqrt{x^2} = |x|$$



AM 25 Se consideră funcția $f : \mathbb{R} \rightarrow \mathbb{R}$, definită prin $f(x) = \ln(1 + x^2)$.

Calculați

$$\lim_{x \rightarrow 1} \frac{f(x) - f(1)}{x - 1}.$$

$$\begin{aligned} \lim_{x \rightarrow 1} \frac{f(x) - f(1)}{x - 1} &= \lim_{x \rightarrow 1} \frac{\ln(1 + x^2) - \ln 2}{x - 1} = \lim_{x \rightarrow 1} \frac{1}{x - 1} \ln \frac{1 + x^2}{2} = \lim_{x \rightarrow 1} \ln \left(\frac{1 + x^2}{2} \right)^{\frac{1}{x-1}} = \\ &= \ln \lim_{x \rightarrow 1} \left(1 + \frac{1 + x^2}{2} - 1 \right)^{\frac{1}{x-1}} = \ln \lim_{x \rightarrow 1} \left(1 + \frac{x^2 - 1}{2} \right)^{\frac{2}{x^2 - 1} \cdot \frac{x^2 - 1}{2} \cdot \frac{1}{x-1}} = \ln e^{\lim_{x \rightarrow 1} \frac{x^2 - 1}{2(x-1)}} = \ln e^{\lim_{x \rightarrow 1} \frac{x+1}{2}} = \\ &= \ln e^{\frac{1+1}{2}} = \ln e = 1 \end{aligned}$$

C



AM 26 * Să se calculeze limita $\lim_{x \rightarrow 0} \frac{\operatorname{tg} x}{x}$.



AM 27 * Să se calculeze

$$\lim_{x \rightarrow 0} \sin \frac{1}{x}.$$



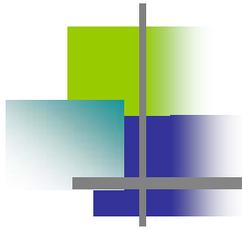
AM 28 * Fie funcția $f : (0, +\infty) \rightarrow \mathbb{R}$, definită prin $f(x) = x^x$. Calculați

$$\lim_{x \searrow 0} x^x.$$



AM 29 * Fie $n \in \mathbb{N}^*$ fixat. Să se calculeze

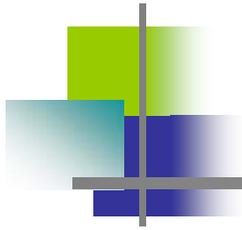
$$\lim_{x \rightarrow 1} \frac{x + x^2 + \dots + x^n - n}{x - 1}.$$



Elemente de analiză matematică

molnar1956@yahoo.com

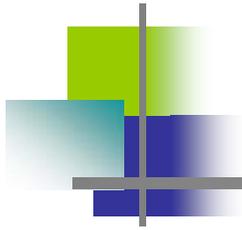




Elemente de analiză matematică

molnar1956@yahoo.com

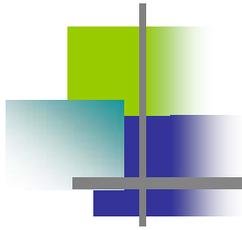




Elemente de analiză matematică

molnar1956@yahoo.com

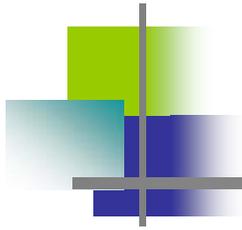




Elemente de analiză matematică

molnar1956@yahoo.com

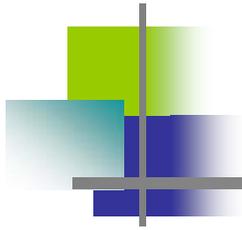




Elemente de analiză matematică

molnar1956@yahoo.com

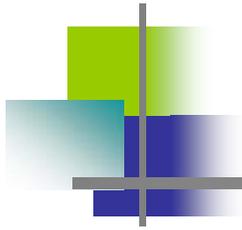




Elemente de analiză matematică

molnar1956@yahoo.com





Elemente de analiză matematică

molnar1956@yahoo.com

